Games and Strategy TA 1

PD, Nash Examples, Oligopoly

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Prisoner Dilemma, example

		Player 2 C	Player 2 D
Player 1	C	3, 3	0, 5
	D	5, 0	1, 1

Figure: An example of Prisoner Dilemma

- C $\rightarrow$ cooperate; D $\rightarrow$ defect
- What is the Nash equilibrium?

Prisoner Dilemma, example

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Figure: An example of Prisoner Dilemma

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Prisoner Dilemma, general formulation

		Player 2	
		C	D
Player 1	C	A, A	B, C
	D	C, B	D, D

Figure: The general Prisoner Dilemma

A game is a PD if...

- $C > A$ and $D > B$ i.e. defecting is always the best choice
- $A > D$, i.e. both players would gain from cooperation

Hence, a PD features $C > A > D > B$.
C is the *temptation*, B is the *sucker's payoff*.

Nash equilibrium

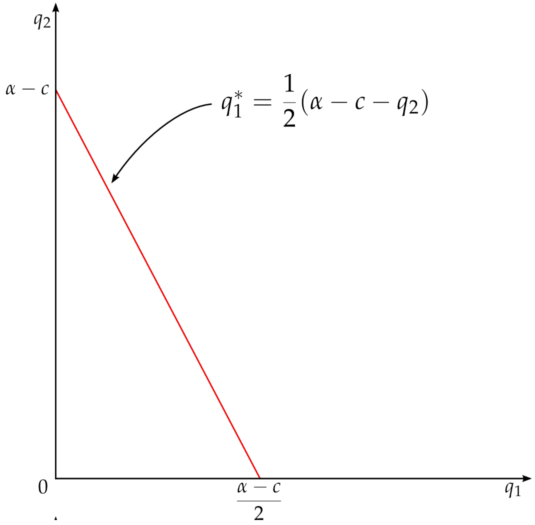
Definition (Nash Equilibrium)

A *Nash Equilibrium* is an action profile a^* with the property that no player i can do better by choosing an action different from a_i^*, given that every other player j adheres to a_j^*. Or, more formally

$$u_i(a^*) \geq u_i(a_i, a^*_{-i}), \ \forall i$$

- in a Nash equilibrium, each player *best responds* to other player's actions
- which are in turn best responses.
- No agent has an incentive to deviate, to change his action from a_i^*.
- In exercises, we will heavily exploit the *no-deviation* property.

Cournot Oligopoly - Graphic



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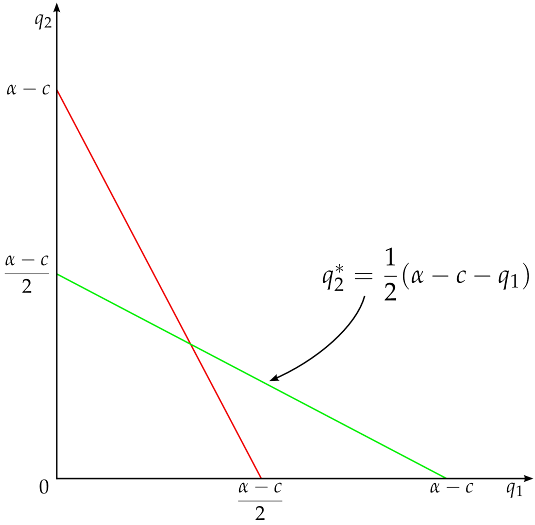

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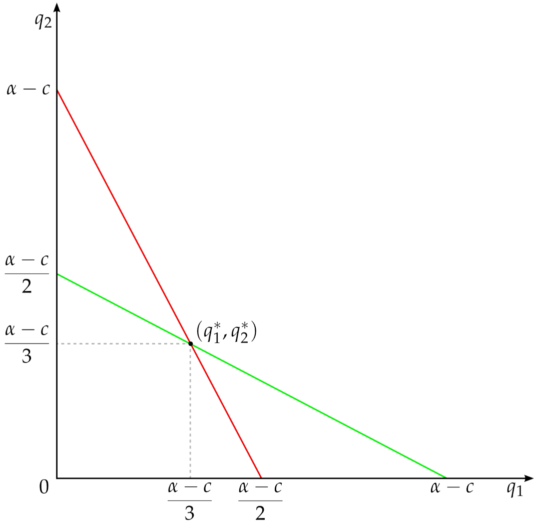

Cournot Oligopoly - 2, 1, many!

	Firms	q_i	Q	P	Π_i
Oligopoly (C)	2	$\frac{\alpha - c}{3}$	$\frac{2(\alpha - c)}{3}$	$\alpha - \frac{2(\alpha - c)}{3}$	$\left(\frac{\alpha - c}{3}\right)^2$

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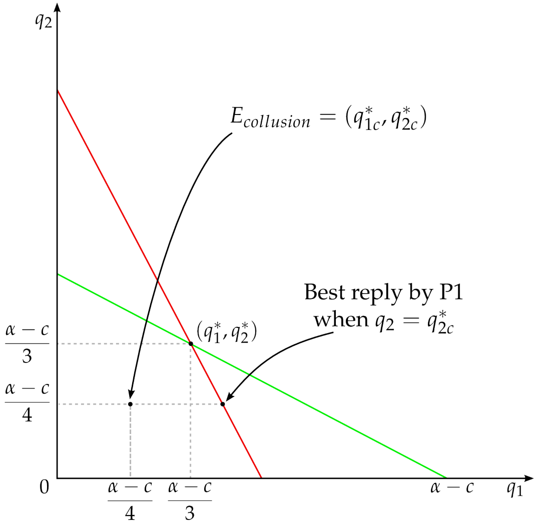
Monopoly	1	$\frac{\alpha - c}{2}$	$\frac{\alpha - c}{2}$	$\alpha - \frac{\alpha - c}{2}$	$\left(\frac{\alpha - c}{2}\right)^2$

Why collusion not Nash? - Graphic



Cournot Oligopoly - 2, 1, many!

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Monopoly	1	$\frac{\alpha - c}{2}$	$\frac{\alpha - c}{2}$	$\alpha - \frac{\alpha - c}{2}$	$\left(\frac{\alpha - c}{2}\right)^2$
Many	n	$\frac{1}{n+1}(\alpha - c)$	$\frac{n}{n+1}(\alpha - c)$	$\alpha - \frac{n}{n+1}(\alpha - c)$	$\left(\frac{\alpha - c}{n}\right)^2$
As $n \to \infty$	∞	$\to 0$	$\to \alpha - c$	$\to c$	$\to 0$