Games and Strategy TA 2

Oligopoly cont'd, Mixed Strategies

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Mixed strategies

- We allow players to *randomize*, i.e. to play a strategy with a certain probability p.
- Players are allowed to choose a *probability distribution* over actions
- And we call this distribution a *mixed strategy*.
- Note that *pure strategies* are just particular cases of mixe strategies,
- in which an action occurs with probability 1 and all others with zero probability.

In 2×2 games, we denote strategy profiles with (p, q), where p is the probability that player 1 plays her first action (as it is shown in the payoff matrix) and q is the probability that player 2 plays her first action. No other element is needed, as the other actions will be played with probability $1 - p$ and $1 - q$.

Matching Pennies

Player 2

		H	T
Player 1	H	1, −1	−1, 1
	T	−1, 1	1, −1

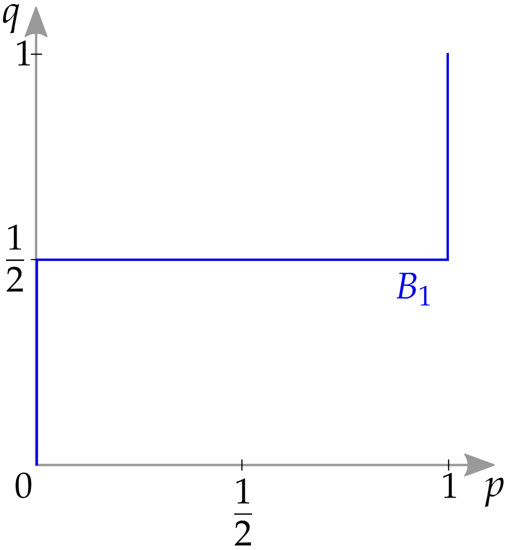
Figure: Matching Pennies

Matching Pennies

			Player 2	
			q	$1-q$
			H	T
Player 1	p	H	$1, -1$	$-1, 1$
	1-p	T	$-1, 1$	$1, -1$

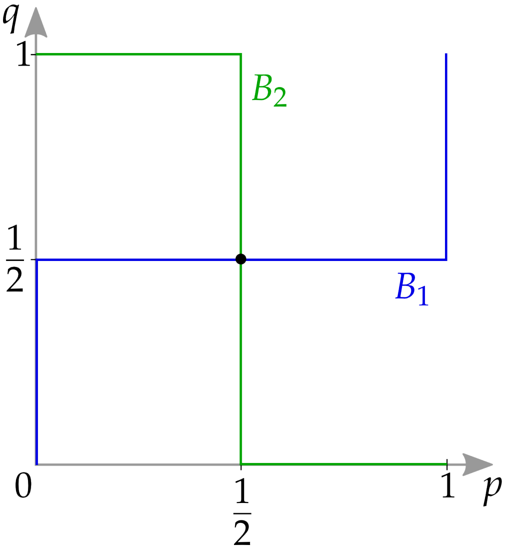
Figure: Matching Pennies

Matching pennies, best responses graph



$$B_1(q) = \begin{cases} 0 & \text{if } q < \frac{1}{2} \\ 0 < p < 1 & \text{if } q = \frac{1}{2} \\ 1 & \text{if } q > \frac{1}{2} \end{cases}$$

Matching pennies, best responses graph



$$B_1(q) = \begin{cases} 0 & \text{if } q < \frac{1}{2} \\ 0 < p < 1 & \text{if } q = \frac{1}{2} \\ 1 & \text{if } q > \frac{1}{2} \end{cases}$$

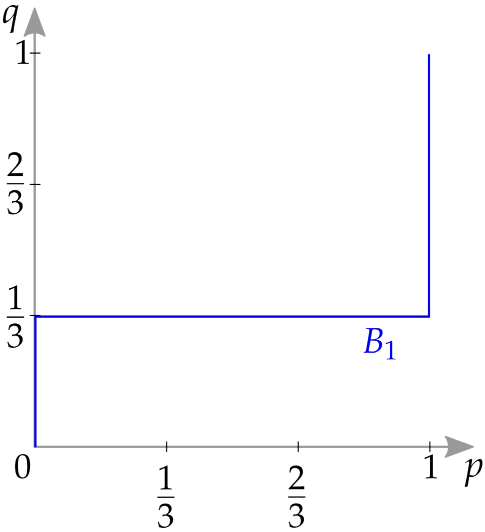
$$B_2(p) = \begin{cases} 1 & \text{if } p < \frac{1}{2} \\ 0 < q < 1 & \text{if } p = \frac{1}{2} \\ 0 & \text{if } p > \frac{1}{2} \end{cases}$$

BoS

		Player 2	
		B	S
Player 1	B	2, 1	0, 0
	S	0, 0	1, 2

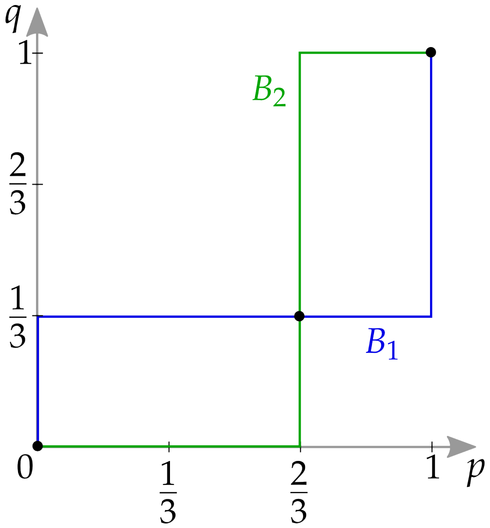
Figure: BoS - Bach or Stravinsky

BoS, best response graph



$$B_1(q) = \begin{cases} 0 & \text{if } q < \frac{1}{3} \\ 0 < p < 1 & \text{if } q = \frac{1}{3} \\ 1 & \text{if } q > \frac{1}{3} \end{cases}$$

BoS, best response graph



$$B_1(q) = \begin{cases} 0 & \text{if } q < \frac{1}{3} \\ 0 < p < 1 & \text{if } q = \frac{1}{3} \\ 1 & \text{if } q > \frac{1}{3} \end{cases}$$

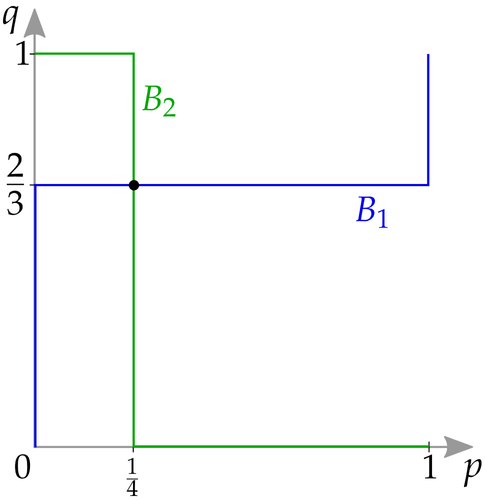
$$B_2(p) = \begin{cases} 0 & \text{if } p < \frac{2}{3} \\ 0 < q < 1 & \text{if } p = \frac{2}{3} \\ 1 & \text{if } p > \frac{2}{3} \end{cases}$$

Example 1

		Player 2: B	Player 2: S
Player 1	B	6, 0	0, 6
	S	3, 2	6, 0

Figure: Example 1 - a game with no pure strategy equilibria

Example 1, best response graph



$$B_1(q) = \begin{cases} 0 & \text{if } q < \frac{2}{3} \\ 0 < p < 1 & \text{if } q = \frac{2}{3} \\ 1 & \text{if } q > \frac{2}{3} \end{cases}$$

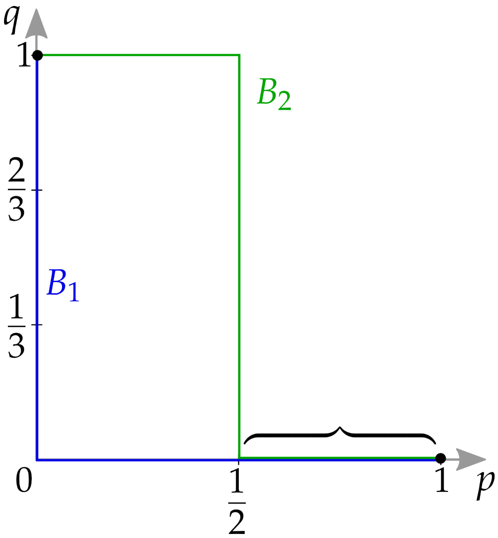
$$B_2(p) = \begin{cases} 1 & \text{if } p < \frac{1}{4} \\ 0 < q < 1 & \text{if } p = \frac{1}{4} \\ 0 & \text{if } p > \frac{1}{4} \end{cases}$$

Example 2

		Player 2: B	Player 2: S
Player 1	B	0, 1	0, 2
	S	2, 2	0, 1

Figure: Example 2 - a game with two pure strategy equilibria

Example 2, best response graph



$$B_1(q) = \begin{cases} 0 < p < 1 & \text{if } q = 0 \\ 0 & \text{if } q > 0 \end{cases}$$

$$B_2(p) = \begin{cases} 1 & \text{if } p < \frac{1}{2} \\ 0 < q < 1 & \text{if } p = \frac{1}{2} \\ 0 & \text{if } p > \frac{1}{2} \end{cases}$$