Games and Strategy TA 3

Extensive Games, Stackelberg Oligopoly, Ultimatum Game

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Recap: sequential games

In extensive games we allow for players to take decisions in *turns*; the games describe the sequence of moves explicitly. An extensive game is said to be *with complete information* if all players know all the actions played before their turn(s). An extensive games is made up of the following elements:

1. A set of Players;
2. A set of *Terminal Histories*;
3. A *Player function* that assigns a player to any non-terminal history;
4. Preferences for the players

Terminal histories are sequences of events that lead from the start to one of the possible ends of a game; a player assigned by the player function to a history h is the player who takes action after h.

Note that any extensive game can be represented in strategic form (i.e., through the payoff matrix).

Entry game: sequential form

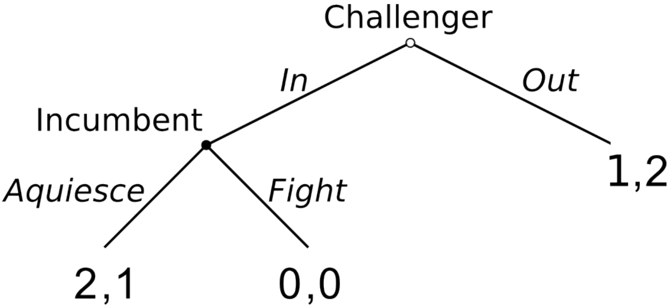

Figure: The entry game

Entry game: strategic form

		Incumbent	
		Acquiesce	Fight
Challenger	In	$2, 1$	$0, 0$
	Out	$1, 2$	$1, 2$

Figure: The strategic form of the entry game

Entry game: strategic form

		Incumbent	
		Acquiesce	Fight
Challenger	In	2, 1	0, 0
	Out	1, 2	1, 2

Figure: The Nash equilibria of the entry game

Subgame perfect equilibrium

Strategies

A player's strategy specifies the action the player chooses for every history after which it is her turn to move. A strategy can be thought as a contingent plan: *if you reach history h_1, do a_1; if h_2, do a_2....* Note that:

1. A strategy specifies also actions in nodes that, following the strategy, would never be reached;
2. A strategy can be thought as a complete set of instructions that the player gives to an agent that plays on her behalf.
3. *In h_1 you should play a_1, in h_2, a_2; but if you made a mistake and in h_1 you chose a_2, then when reaching h_3, do play a_3, etc...*

Subgames

For any non-terminal history h, a subgame is the part of the game that remains after h has been reached. Note that this includes the whole game itself, since also the empty history is a non-temrinal history.

Subgame perfect equilibrium

A subgame perfect equilibrium is a strategy profile s^* such that in no subgame can any player i do better by choosing a strategy different from s_i^*, given that any other player j adheres to s_j^*.

Entry game: subgames

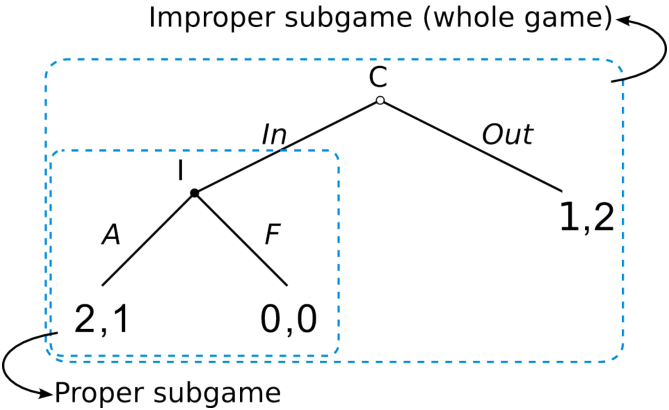

Figure: The subgames of the entry game

Backward Induction

To find Subgame Perfect Equilibria of any *finite* game, it is possible to reason by *backward induction*, i.e. to start reasoning from the end of the game, find out what players will do if that particular node is reached, and then work backwards to the game tree until the empty (initial) history is reached.

Formally, defining the *length of a subgame* as the longest history in the subgame, when using backward induction we first analyse the outcome of the subgames of length 1, then of length 2, and so on until the beginning of the game. DOing so we can then build an optimal strategy for each player.

If every player moving at the start of every subgame has one and only one optimal action, then the set of equilibria identified through backward induction is equal to the set of subgames perfect equilibria.

Entry game: backward induction

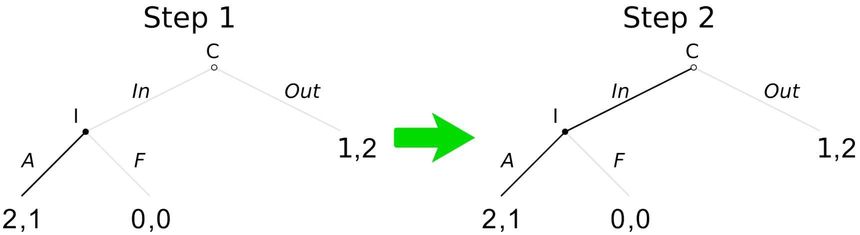

Figure: Backward Induction in the entry game

Army game: keeping the bridge

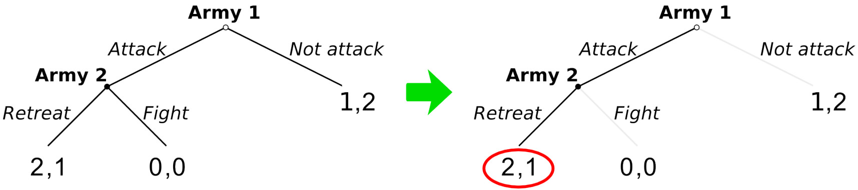

Figure: The army game: non-credible threat

Army game: burning the bridge

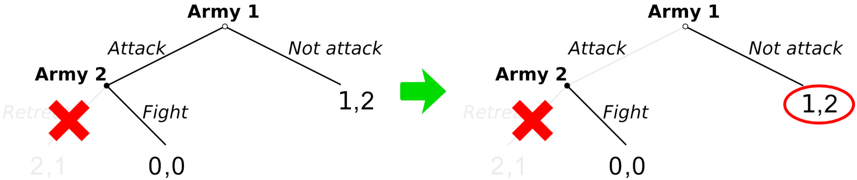

Figure: The army game: strength through precommitment

Stackelberg Oligopoly

	Firms	q_i	Q	P	Π_i
Cournot	2	$\frac{\alpha - c}{3}$	$\frac{2(\alpha - c)}{3}$	$\alpha - \frac{2(\alpha - c)}{3}$	$\left(\frac{\alpha - c}{3}\right)^2$
Stackelberg Leader	2	$\frac{\alpha - c}{2}$	$\frac{3(\alpha - c)}{4}$	$\frac{\alpha + 3c}{4}$	$\frac{(\alpha - c)^2}{8}$
Stackelberg Follower	2	$\frac{\alpha - c}{4}$	$\frac{3(\alpha - c)}{4}$	$\frac{\alpha + 3c}{4}$	$\frac{(\alpha - c)^2}{16}$

Ultimatum Game

The game

Two persons must share a pie of dimension c. The first person (player 1) is asked to offer to the second (2) a positive amount of the pie, $0 < x \leq c$. Player 2 can Accept or Refuse the offer. If the offer is accepted, the pie is divided accordingly; if it is rejected, both players get zero. The game can be represented by:

Players: 1, 2

Terminal Histories: all sequences (x, Z) where $0 < x \leq c$, and $Z = A, R$

Players function: $P(\varnothing) = 1$, $P(x) = 2\,, \forall x$.

Payoffs: if (x, A), $\Pi = (x - c, x)$; if (x, R), $\Pi = (0, 0)$.

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Nash: every accepted offer is a Nash Equilibrium.

Subgame Perfect: Player 1 offer the lowest amount possible, player two accepts.

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But experiments tell a very different story...