Games and Strategy TA 5
Bargaining

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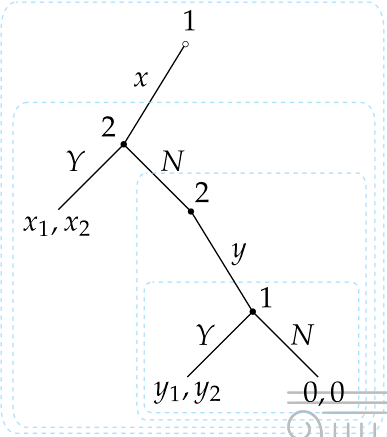
Barganing as an extensive game, finite horizon

Finite horizon

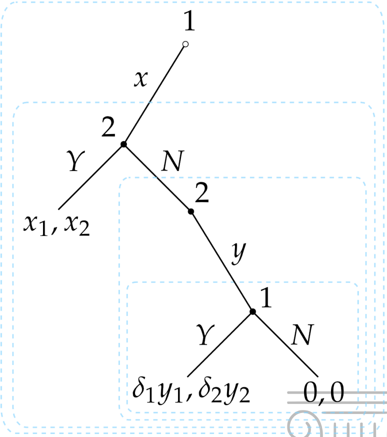
- We can apply backward induction;
- Identical to ultimatum game;
- last proposer has all powers;
- last responder powerless;
- This is true no matter how long the game.



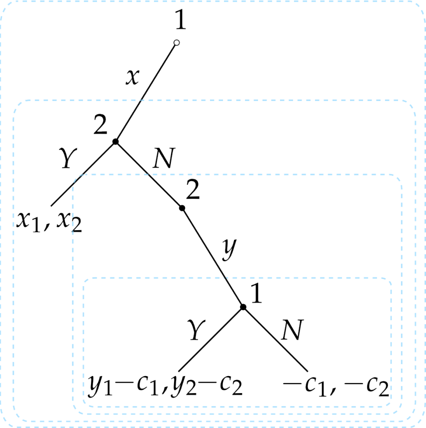
Barganing as an extensive game, finite horizon with δ

Finite horizon with δ

- We can apply backward induction;
- Players are impatient, $0 < \delta < 1$
- Last responder can exploit the opponent's δ
- Offering in the first period exactly that δ
- In the game here, in SPE 1 offers $(1 - \delta_2, \delta_2)$, 2 accepts, and 2 offers $(0, 1)$ in histories in which she gets to make an offer.



Exercise 468.1: constant cost of delay



Finite horizon with c_i

- We can apply backward induction;
- Players have fixed cost of delay c_i
- Last responder can exploit the opponent's c_i
- Offering in the first period $1 - c_i$
- In the game here, in SPE 1 offers $(c_2, 1 - c_2)$, 2 accepts, and 2 offers $(0, 1)$ in histories in which she gets to make an offer.

Barganing as an extensive game, infinite horizon

Infinite horizon

- We cannot use backward induction
- Since there is not a last responder, the two players tend to have equal power
- We assume players to have discount factor δ_i
- Game is stationary: every subgame looks exactly as the game itself (with discounted payoffs)

Solution

- We look for a candidate stationary equilibrium: each player has a simple rule implying offering always the same amount and accepting all offers exceeding a threshold.
- We further conjecture that in equilibrium all offers are accepted.

Solution

- Following this logic, the solution turns out to be,if player 1 proposes x and player 2 proposes y

$$x^* = \left(\frac{1-\delta_2}{1-\delta_1\delta_2}, \frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}\right), y^* = \left(\frac{\delta_1(1-\delta_2)}{1-\delta_1\delta_2}, \frac{1-\delta_1}{1-\delta_1\delta_2}\right)$$

- Which Rubinstein proves to be the only SPE of the game.

Recap: Nash bargaining solution

An axiomatic approach

The strategic approach sets up the problem as a game, and solves the game looking for equilibria. The *axiomatic* approach sets up the problem, and looks for 'reasonable' properties that the solution should feature; it then moves on to find the solution(s) that possess those same properties

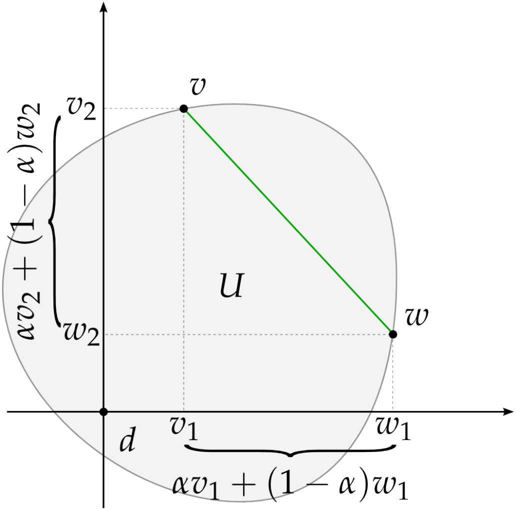
A bargaining problem between two players is composed of:

1. A set of the Bernoulli utilities over feasible alternatives $\mathcal{U} = \{(v_1, v_2) : u_1(x) = v_1, u_2(x) = v_2, \forall\, x \in X\}$
2. A disagreement outcome (status quo, $(0, 0)$, or worse), $d = (u_1(d), u_2(d))$

We need the set $\mathcal{U}$ to have the following properties

- $d \in \mathcal{U}$;
- $\exists (v_1, v_2)$ such that $v_1 > d_1, v_2 > d_2$
- $\mathcal{U}$ convex
- $\mathcal{U}$ compact, i.e. bounded and closed.

Convexity of $\mathcal{U}$

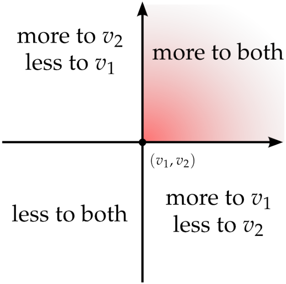


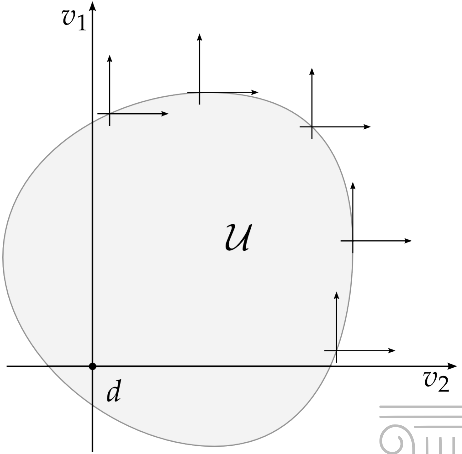
Axioms

Nash proposed the following four axioms:

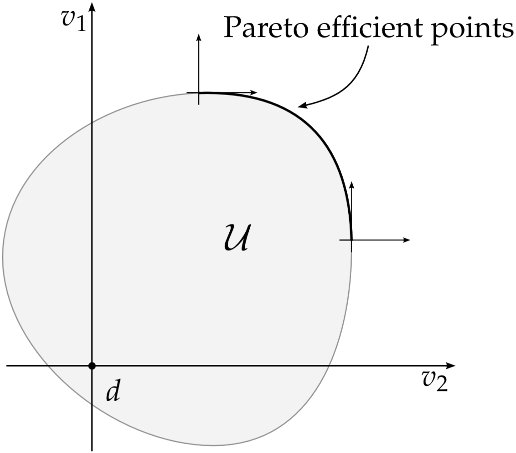
- PAR: The solution should be Pareto-efficient
- SYM: If the problem (players, $\mathcal{U}$, d) is symmetric, so should be the solution;
- INV: The problem is invariant to linear transformations of the utility functions
- IIA: If the solution in a large $\mathcal{U}'$ is within a subset $U \subset U'$, then tha same solution holds for $\mathcal{U}$.

Axiom 1: Pareto efficiency (PAR), I

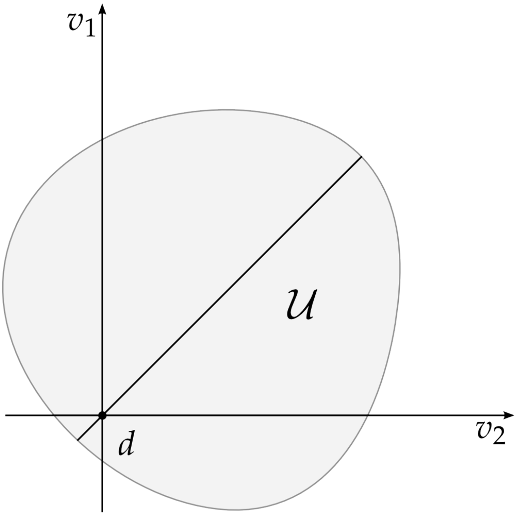




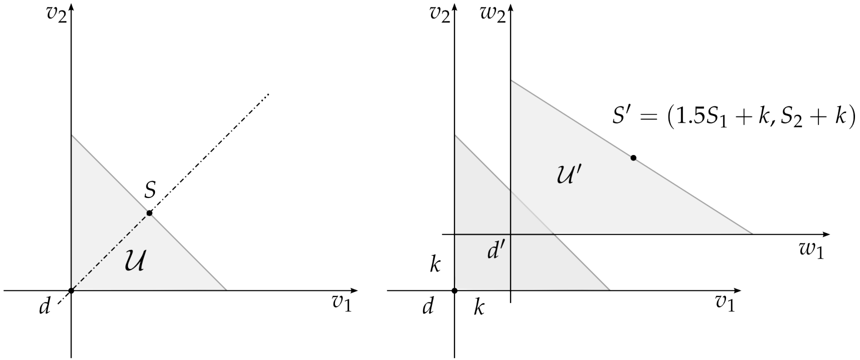
Axiom 1: Pareto efficiency (PAR), II



Axiom 2: Symmetry (SYM)



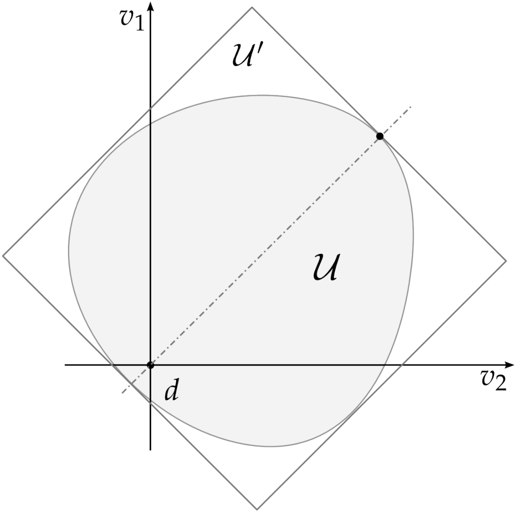
Axiom 3: Invariance to equivalent payoff representations (INV)



Linear transformation of utility function:

$w_1 = 1.5v_1 + k \quad w_2 = v_2 + k$

Axiom 4: Independence of irrelevant alternatives (IIA)



Exercise 486.1: *PAR*, *SYM*, *IIA*