Games and Strategy TA 6
Bargaining cont'd

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Recap: bargaining

A bargaining problem between two players is composed of:

1. A set of the Bernoulli utilities over feasible alternatives
   $\mathcal{U} = \{(v_1, v_2) : u_1(x) = v_1, u_2(x) = v_2, \forall\, x \in X\}$
2. A disagreement outcome (status quo, $(0, 0)$, or worse), $d = (u_1(d), u_2(d))$

We need the set $\mathcal{U}$ to have the following properties

- $d \in \mathcal{U}$;
- $\exists (v_1, v_2)$ such that $v_1 > d_1, v_2 > d_2$
- $\mathcal{U}$ convex
- $\mathcal{U}$ compact, i.e. bounded and closed.

A Bargaining problem is a set $(\mathcal{U}, d)$; a solution is $f : (\mathcal{U}, d) \mapsto \mathbb{R}^2$

Nash Axioms

Nash proposed the following four axioms:

- PAR: The solution should be Pareto-efficient
- SYM: If the problem (players, $\mathcal{U}$, d) is symmetric, so should be the solution;
- INV: The problem is invariant to linear transformations of the utility functions
- IIA: If the solution in a large $\mathcal{U}'$ is within a subset $U \subset U'$, then the same solution holds for $\mathcal{U}$.

Nash Solution

Nash showed that only one f satisfies his four axioms:

The solution will be the amount that maximises the product of the difference between earnings and the disagreement outcome; that is

$$f^N(\mathcal{U}, d) = \arg\max(u_1(x_1) - d_1)(u_2(x_2) - d_2)$$

In the special case in which $(d_1, d_2) = (0, 0)$, and with two players, it boils down to

$$f^N(\mathcal{U}, d) = \arg\max(u_1(x_1))(u_2(x_2))$$

- Note that the utility functions are Bernoulli - i.e. preferences over lotteries over the set of feasible alternatives X;
- This means that risk aversion plays a role.

Recap: risk aversion

- In the context of risky choices, i.e. of lotteries, player's attitudes to risk matter
- Let's imagine to have a binary choice between two lotteries
- Lottery A gives 100$ for sure (with probability 1)
- Lottery B gives 200$ with probability 0.5 and 0 with probability 0.5

1. Same expected value $EV = \sum_i p_i \cdot w_i$; $EV_A = EV_B = 100$
2. Different variances $VAR = \sum_i p_i \cdot (w_i - EV)^2$; $VAR_A = 0$, $VAR_B = 10000$

A risk-averse player, *EV* being equal, preferes the lottery with lower *VAR*

Utility diagram - *risk-neutral*

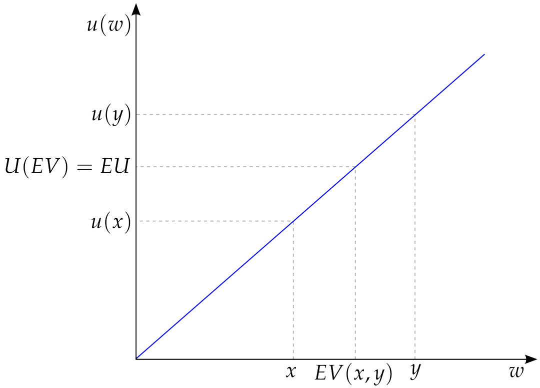


Figure: Risk-neutral: $U(EV) = EV(U)$

Utility diagram - *risk-averse*

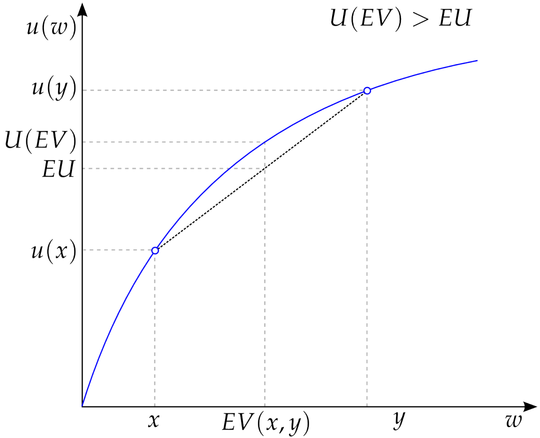
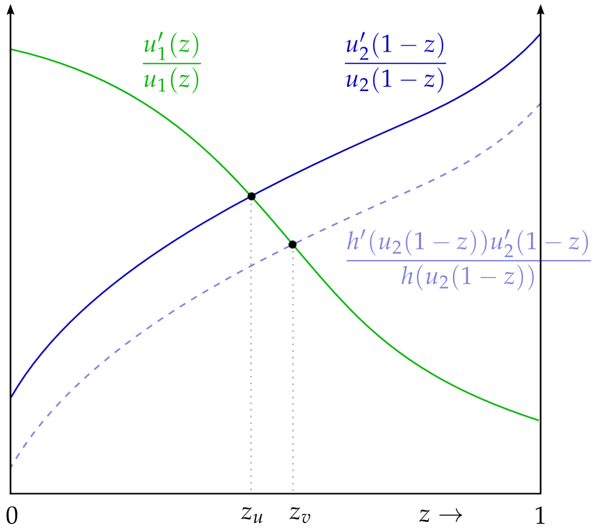


Figure: Risk-averse: $U(EV) > EV(U)$

Splitting a dollar with risk-aversion



Problems with IIA

- Intuitively weak
- Useful for some bargaining practices, but not for others
- Repeatedly refuted experimentally (Allais paradox)

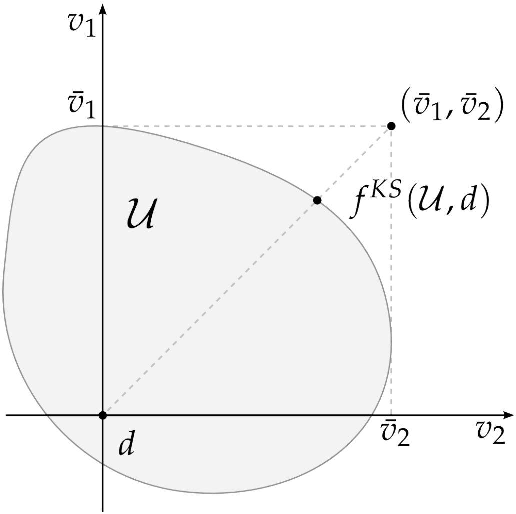
Allais paradox: the problem

Experiment 1				Experiment 2			
Lottery A		**Lottery B**		**Lottery A**		**Lottery B**	
Win	Prob	Win	Prob	Win	Prob	Win	Prob
1 million	89 %	1 million	89 %	0	89 %	0	89 %
1 million	11 %	0	1 %	1 million	11 %	0	1 %
		5 millions	10 %			5 millions	10 %

Allais paradox: solution

Experiment 1				Experiment 2			
Lottery A		Lottery B		Lottery A		Lottery B	
Win	Prob	Win	Prob	Win	Prob	Win	Prob
1 million	89 %	1 million	89 %	0	89 %	0	89 %
1 million	11 %	0	1 %	1 million	11 %	0	1 %
		5 millions	10 %			5 millions	10 %

KS solution: graphics



KS *vs* Nash problem

Find the Nash and the KS solutions of the bargaining problem $(\mathcal{U}, d)$ in which $\mathcal{U}$ is the triangle with corners at $(0,0), (1,1)$ and $(2,0)$, and $d = (0,0)$.