Problem Set I: Preferences, W.A.R.P., consumer choice

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Exercises will be solved in class on *18th January 2010*

1. MWG, Exercise 1.B.1 + 1.B.2: properties of $\succsim$

Prove that if $\succsim$ is rational (complete and transitive), then

1. $\succ$ is both irreflexive ($x \succ x$ never holds) and transitive (if $x \succ y$ and $y \succ z$, then $x \succ z$);
2. $\sim$ is reflexive ($x \sim x$, $\forall x$), transitive (if $x \sim y$ and $y \sim z$, then $x \sim z$) and symmetric (if $x \sim y$ then $y \sim x$);
3. if $x \succ y \succsim z$ then $x \succ z$.

2. MWG 1.B.3 + 1.B.4.: $\succsim$ and $u(\cdot)$

- Show that if $f : \mathbb{R} \mapsto \mathbb{R}$ is a strictly increasing function and $u : X \mapsto \mathbb{R}$ is a utility function representing the preference relation $\succsim$, then the function $v : X \mapsto \mathbb{R}$ defined by $v(x) = f(u(x))$ is also a utility function representing $\succsim$;
- Consider a prefernce relation $\succsim$ and a function $u : X \mapsto \mathbb{R}$. Show that if $u(x) = u(y)$ implies $x \sim y$ and if $u(x) > u(y)$ implies $x \succ y$ then $u(\cdot)$ is a utility function representing $\succsim$.

3. MWG 1.B.5: $\succsim$ and $u(\cdot)$, II

Show that if X is finite and $\succsim$ is a rational preference relation on X, then there is a utility function $u : X \mapsto \mathbb{R}$ that represents $\succsim$.

4. Exercise on W.A.R.P.

Consider a choice problem with choice set $X = \{x, y, z\}$. Consider the following choice structures:

- $(\mathcal{B}', C(\cdot))$, in which $\mathcal{B}' = \{\{x, y\}, \{y, z\}, \{x, z\}, \{x\}, \{y\}, \{z\}\}$ and $C(\{x, y\}) = \{x\}$, $C(\{y, z\}) = \{y\}$, $C(\{x, z\}) = \{z\}$, $C(\{x\}) = \{x\}$, $C(\{y\}) = \{y\}$, $C(\{z\}) = \{z\}$.
- $(\mathcal{B}'', C(\cdot))$, in which $\mathcal{B}'' = \{\{x, y, z\}, \{x, y\}, \{y, z\}, \{x, z\}, \{x\}, \{y\}, \{z\}\}$ and $C(\{x, y, z\}) = \{x\}$, $C(\{x, y\}) = \{x\}$, $C(\{y, z\}) = \{z\}$, $C(\{x, z\}) = \{z\}$, $C(\{x\}) = \{x\}$, $C(\{y\}) = \{y\}$, $C(\{z\}) = \{z\}$.
- $(\mathcal{B}''', C(\cdot))$, in which $\mathcal{B}''' = \{\{x, y, z\}, \{x, y\}, \{y, z\}, \{x, z\}, \{x\}, \{y\}, \{z\}\}$ and $C(\{x, y, z\}) = \{x\}$, $C(\{x, y\}) = \{x\}$, $C(\{y, z\}) = \{y\}$, $C(\{x, z\}) = \{x\}$, $C(\{x\}) = \{x\}$, $C(\{y\}) = \{y\}$, $C(\{z\}) = \{z\}$.

For every choice structure say if the WARP is satisfied and if it exists a rational preference relation $\succsim$ that rationalizes $C(\cdot)$ relative to *its* $\mathcal{B}$. If such a rationalization is possible, write down. Comment on your results.

5. MWG 1.D.2: $\succsim$ and W.A.R.P.

Show that if X is finite, then any rational preference relation generates a nonempty choice rule; that is, $C(B) \neq \varnothing$ for any $B \subset X$ with $B \neq \varnothing$.