Problem Set III: Demand, Comparative static, monotonicity

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Exercises will be solved in class on *February 3rd, 2010*

1. MWG 2.F.10: substitution matrix

Consider the following demand function:

$$x_1(p,w) = \frac{p_2}{p_1 + p_2 + p_3}\frac{w}{p_1}$$

$$x_2(p,w) = \frac{p_3}{p_1 + p_2 + p_3}\frac{w}{p_2}$$

$$x_3(p,w) = \frac{p_1}{p_1 + p_2 + p_3}\frac{w}{p_3}$$

1. Compute the substitution matrix. Show that at $p = (1,1,1)$ it is negative semidefinite but not symmetric.
2. Show that this demand function does not satisfy the weak axiom. [Hint: consider $p = (1,1,\varepsilon)$ and show that the matrix is not negative semidefinite for $\varepsilon > 0$ small].

2. MWG 2.F.17: demand

In an L-commodity world, a consumer's Walrasian demand function is

$$x_k(p,w) = \frac{w}{\sum_{l=1}^{L} p_l}, \text{for } k = 1,\dots,L$$

1. Is this demand homogeneous of degree zero in (p,w)?
2. Does it satisfy Walras' Law?
3. Does it satisfy the Weak Axiom?
4. Compute the Slutsky substitution matrix for this demand function. Is it negative semidefinite? Negative definite? Symmetric?

3. MWG 3.B.2: monotonicity

The preference relation $\succsim$ defined on the consumption set $X = \mathbb{R}^L_+$ is said to be *weakly monotone* if and only if $x \geq y$ implies $x \succsim y$. Show that if $\succsim$ is transitive, locally non satiated and weakly monotone, then it is monotone.

4. UMP with Cobb-Douglas utility

Let the utility function of a consumer be $U = x_1^{\alpha} x_2^{1-\alpha}$, and its budget constraint be $x_1 p_1 + x_2 p_2 \leq w$. Solve the Utility Maximisation Problem (UMP) and derive the demand functions $x_i(p,w)$, $i = 1,2$ and the indirect utility function $v(p,w)$.