Problem Set VI: Edgeworth Box

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Exercises will be solved in class on *March 15th, 2010*

1. Edgeworth box

Consider a pure-exchange, private-ownership economy, consisting in two consumers, denoted by $i = 1, 2$, who trade two commodities, denoted by $l = 1, 2$. Each consumer i is characterized by an endowment vector, $\omega_i \in \mathbb{R}^2_+$, a consumption set, $X_i = \mathbb{R}^2_+$, and regular and continuous preferences, $\succsim_i$ on X_i.

1. Assuming that the consumers' endowments are $\omega_1 = (1, 2)$ and $\omega_2 = (2, 1)$, respectively, construct the Edgeworth Box relative to economy under consideration. With reference to the same economy, define the following notions: competitive equilibrium, Pareto-efficient allocation, Pareto set, contract curve.

2. Find the equation describing the Pareto set (internal solutions); then, taking commodity 1 as the numeraire, hence positing $p_1 \equiv 1$, find the competitive equilibrium allocation and price system; and, finally, draw your results in the Edgeworth Box in each of the following two cases:

   (a) both consumers' preferences are represented by the same Cobb-Douglas utility function: $u_i(x_{1i}, x_{2i}) = x_{1i}^{\frac{1}{3}} x_{2i}^{\frac{2}{3}}$;

   (b) the two consumers' preferences are respectively represented by the following quasilinear utility functions: $u_1(x_{11}, x_{21}) = x_{11} + \ln x_{21}$; $u_2(x_{12}, x_{22}) = x_{12} + 2 \ln x_{22}$.

3. Explain in which of the above two cases the preferences are homothetic and in which case, instead, the preferences are such as to rule out the wealth effects usually associated with price changes. How such peculiar properties of consumers' preferences affect the Pareto set and the shadow prices, i.e., the prices implicit in the Pareto-efficient allocations?

2. MWG 15.B.1 + 15.B.3: Edgeworth box

Consider an Edgeworth box economy with two goods and two consumers with locally non satiated preferences. Let $x_{li}(p)$ be consumer i's demand of good l at prices $p = (p_1, p_2)$.

1. Show that $p_1(\sum_i x_{1i}(p) - \bar{\omega}_1) + p_2(\sum_i x_{2i}(p) - \bar{\omega}_2) = 0$ for all prices p.

2. Argue that if the market for good 1 clears at prices $p^* \gg 0$, then so does the market for good 2; hence, p^* is a Walrasian equilibrium price vector.

3. Argue graphically that the Walrasian equilibrium is Pareto optimal.

Next week we will be holding a classrom experiment on bilateral trading.