Problem Set VII: Edgeworth Box, Robinson Crusoe

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Exercises will be solved in class on *March 22nd, 2010*

1. Edgeworth box

Consider a pure-exchange, private-ownership economy, consisting in two consumers, denoted by $i = 1, 2$, who trade two commodities, denoted by $l = 1, 2$. Each consumer i is characterized by an endowment vector, $\omega_i \in \mathbb{R}^2_+$, a consumption set, $X_i = \mathbb{R}^2_+$, and regular and continuous preferences, $\succsim_i$ on X_i. Initial endowments are given by $\omega_1 = (4, 2)$ and $\omega_2 = (2, 3)$. Individual utility functions are $u_1(x_{11}, x_{21}) = x_{11}x_{21}$ and $u_2(x_{12}, x_{22}) = x_{12} + x_{22}$.

1. Draw the Edgeworth Box for this economy, drawing endowment point ω and the indifference curves passing through it for both consumers.
2. Find analytically the Pareto Set (interior points and, separately, boundary points) and the Contract curve. Draw them both in the Edgeworth Box.
3. Find the competitive equilibrium prices and alocations. Draw it in the Edgeworth Box.
4. Take the point $\tilde{x} = ((\tilde{x}_{11}, \tilde{x}_{21}), (\tilde{x}_{12}, \tilde{x}_{22})) = ((1, 1), (5, 4))$. Draw it in the Edgeworth box, show that it is in the Pareto set, and write down the implicit shadow prices. Compute the transfers T_1 and T_2, with $T_1 + T_2 = 0$, that ought to be assigned to either consumer in order that, starting from the initial endowments, the allocation concerned can be obtained as a price equilibrium with transfers.

2. Robinson Crusoe Economy

Consider a ”Robinson Crusoe economy”, i.e., a private-ownership, competitive economy with only one consumer ($I = 1$), one producer ($J = 1$), and two commodities ($L = 2$). The producer is characterized by a single-output technology, with production set $Y = \{(-z, q) \in \mathbb{R}^2 \mid q - f(z) \leq 0 \text{ and } z \geq 0\}$, where z is the quantity of input (”labor time”), q is the quantity of output (”consumers' good”), and $f(z) = 2z$ is the production function. The consumer is characterized by a consumption set $X = \{x = (x_1, x_2) \in \mathbb{R}^2_+\}$, where x_1 and x_2 are the quantities of ”leisure time” and ”consumers' good”, respectively, and a Cobb-Douglas utility function $u(x_1, x_2) = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$. The consumer owns the endowments $\bar{\omega} = (\bar{L}, 0) \in \mathbb{R}^2_+$, where $\bar{L} = 24$ denotes the time units the consumer has at his disposal (time can be used as either ”leisure time” or ”working time”); the endowment of ”consumers' good” is nil. The consumer owns the producer, thereby getting the whole of the latter's profits. Let p be the price of the ”consumers' good” and w be the wage, i.e., the price of both the ”leisure time” and the ”working time”. Let the ”consumers' good” be the numeraire of the economy, and consequently set $p \equiv 1$.

1. After examining the properties of the production function, focussing particularly on the nature of the returns to scale, write down and solve the producer's profit maximization problem, determining the demand correspondence for ”labor time” and the supply correspondence of the ”consumers' good” (the only argument of both correspondences being the wage w, which coincides with the real wage, $\frac{w}{p}$, since $p \equiv 1$).
2. Find the producer's profit function $\pi(\cdot)$, showing in particular that, given the nature of the returns to scale, such function is not always well-defined: it can either take the value 0 or diverge to $+\infty$. Hence, prove that the value 0 is the only one to be consistent with a competitive equilibrium.
3. After examining the properties of the utility function, write down and solve the consumer-worker's utility maximization problem (for an interior optimum and assuming the producer's profits to be equal to 0, since this is the only value that has been proven to be consistent with a competitive equilibrium), thereby determining the Walrasian demand functions for both ”leisure time” and ”consumers' good” as well as the Walrasian supply function of ”working time” (all having the wage w as the only argument).
4. Illustrate the producer's and the consumer-worker's problems in one and the same diagram.
5. Find the (unique) competitive equilibrium of the ”Robinson Crusoe's economy” and plot it in the above diagram.
6. Prove that the competitive equilibrium allocation is the only Pareto-efficient allocation of the economy.